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Measuring cognitive and non-cognitive systems of reasoning: some preliminary findings

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This paper reports on preliminary findings involving a pilot project for a doctoral thesis by research, which seeks to examine the issues of creativity in problem solving and of how such creativity may be fostered in children under instruction. In particular, the design, trial and statistical appraisal of a new self-report instrument (viz: Systems of Reasoning Questionnaire SRQ) that was formulated to assist in the identification, description and measurement of some cognitive and non-cognitive forms of reasoning is described. The SRQ comprises five scales, which tap into Rule-based and Associative forms of reasoning as characterized by Sloman, 1996. The pilot project involved sampling upper primary and lower secondary school students solving novel mathematics problems within the Mathematics Challenge for Young Australians 2000.

Cognitive, Non-cognitive, Rule-based, Associative, Creativity

INTRODUCTION

Recent critique of educational research related to constructivist thinking, particularly with regard to mathematics education, has pointed to gaps in understanding of the part cognitive and non-cognitive systems of reasoning play in solving problems (Davis 1996, Davis et al 1996, Taylor P.1996, Begg 1999, Gunn 1999). Indeed concern has been expressed that constructivist approaches to mathematics teaching and learning focus largely on cognitive forms of reasoning and take little account of other kinds of knowing such as those of how feeling and emotion are constructed in the learning and/or the problem solving process, of how non-conscious or unformulated knowing may contribute, or of how things may be known intuitively.

Given that for the past twenty five years at least, cognitive psychology (unlike its counterpart of social psychology) has largely ignored affect (Zajonic, 1980, Anderson, 1999) and that many current models of constructivist teaching and learning have their early beginnings in cognitive and experimental psychology (Good, Wandersee & St Julien 1993; Phillips 2000), the concerns mentioned above would appear to be entirely consistent and largely inevitable. Since teaching and learning must of necessity, centre on the total individual, educational practices which focus solely on cognitive aspects to the exclusion of non-cognitive ones, are in the long term, likely to be found wanting (Begg 1999). In fact these concerns have been finding expression in literature for some considerable time. As far back as 1976 for instance, Miller and Johnson-Laird in their notable work on language and perception in which "fearfully cognitive and dispassionate"(p.111) information–processing systems of perception (*Perceive*), memory (*Remember*) and intention (*Intend*) were presented, had this to say about feeling (*Feel*):

> *Feel* is an indispensable predicate for any complete psychology and that it probably lies much closer than *Perceive, Remember*, and *Intend* to the basic sources of energy that keep the whole system running (p.112) (Miller and Johnson-Laird italics)

In 1988, Torrence and Rockstein elaborating on future directions in research and teaching wrote:

> As a first step in these directions, teachers and teacher educators must work together to develop whole-brain units of learning that combine objectives from the cognitive, affective and intuitive domains (p.289)

In 1999, mathematics educator, Begg writing about non-cognitive knowing in the context of research on learning, teaching and curriculum stated:

> Whether we consider emotions as unformulated knowledge, as personal constructions, or as actions, there seems to be a need to consider them not apart from other constructions and actions but as part of our being which is intimately interrelated with our knowing. (p.70)

Perfinking and Creativity

Highlighting the idea that individuals may, perceive, feel and think all at once, Bruner (1986) cites David Krech as coining the term "perfink" and goes on to add that action, is an important component of "perfinking" (Bruner 1990). Indeed, perceiving, feeling, thinking and acting are all likely to be present during learning (Davis 1996). Thus in the endeavour to assist children solve unseen mathematical problems as is the case in novel problem situations, the capacity to draw upon "perfinking" not merely cognitive thinking, may well increase the potential for the enactment of a successful and creative solution.

Certainly teaching children to tap into both their cognitive and non-cognitive forms of reasoning holds significance in the field of creativity research (Russ 1993; Sternberg and O'Hara 1999) where some operational definitions of creativity embrace the notion of both subjective and objective knowing (Taylor 1989). Research in this field has shown that when students are taught and assessed in a way that values both their subjective and objective creative abilities their academic performance improves (Sternberg et al 1996; Sternberg et al 1998). In seeking to make explicit the nature of the creative process Koberg and Bagnall (1976) for example described creativity as:

> … both the art and the science of thinking and behaving with both subjectivity and objectivity. It is a combination of feeling and knowing: of alternating back and forth between what we sense and what we already know. Becoming more creative involves becoming awake to both; discovering a state of wholeness which differs from a primarily objective or subjective person which typifies our society. (p.8)

While much has been written about the importance of creativity to novel problem solving (Sternberg 1999) and terms such as "perfinking" point to gaps in approaches to teaching and learning (Davis 1996), little has been done to address these issues in education generally and in mathematics education in particular. In point of fact, apart from social psychological consideration such as awareness that feelings of anxiety may affect problem solving ability, non-cognitive forms of reasoning have generally gone un-noticed in the teaching of students to solve problems creatively. This is partly because the identification, description and measurement of non-cognitive forms of reasoning in the schooling context have largely been illusive and partly because their interaction with cognitive forms of reasoning remains uncertain. While the creativity literature has many examples of individuals drawing upon the elements of imagination, intuition and feeling in creating eminent works, including mathematicians (Parr, 1974; Morris, 1993; Simonton, 1999; Feist, 1999; Policastro & Gardner 1999) little if any research has been done to uncover these forms of reasoning amongst students under mathematics instruction.

Hence it is the purpose of this research to determine whether cognitive and non-cognitive forms of reasoning can be identified and described in the novel mathematics problem-solving context and whether they can be measured through a self-reporting instrument designed to tap into two kinds of reasoning. The remainder of this paper outlines the methods used, the reasons for selecting these methods and the resulting findings.

METHODOLOGY

In 1993 Biggs presented a model of teaching and learning, (termed the '3P' model of teaching and learning) which provides a method for separating and assessing various elements in a teaching and learning situation. Central to this model is the notion that by measuring a student's approach to a given learning activity it is also possible to tap into other elements of the teaching and learning system such as its presage, process and product (i.e. the three Ps) (Biggs 2001). The presage component may be determined by student factors and by the teaching context, the process component by the nature of the learning activities and the product component, by the learning outcomes (Biggs 2001).

In similar vein, it was conjectured that if it is possible to measure a student's reasoning approaches to a novel problem solving activity, it may also be possible not only to tap into the process(es) (both cognitive and non-cognitive) associated with creative problem solving, but also, to access in the long term other elements of the teaching and learning system which may influence student creativity in a mathematical problem solving context. One analgous model suggests creativity may be determined by the intersecting set of the person, process, product and environment (Aldous 1999, 2000). As Shuell (1986) explained "It is important to remember that what the student does is more important than what the teacher does." (p.429)

Determining the Learning Activity

The research of these possibilities and conjectures clearly required a student learning and problem solving activity on a broad basis. Furthermore the learning activity also needed to be concomitant with a four stage model, of creative problem solving frequently depicted by psychologists, mathematicians and scientists alike (Parr 1974, Russ 1993). (This model was first described in the early writings of Wallas (1926) and Hadamard (1945)). In brief the four stages of creative problem solving include: *preparation*, *incubation*, *illumination* and *elaboration/verification*. In addition the learning activity had the indispensable requirements that it be novel, if creativity were to be expressed (Runco & Sakamoto 1999) and that the activity should be undertaken by a sufficiently wide-ranging sample of students to be statistically both useful and fruitful (Keeves 1997).

Thus the student learning activity central to the framework for research was determined to be the Mathematics Challenge for Young Australians, a national competition of novel mathematics problem solving for upper primary and lower secondary students. The activity, an annual event, has the unique feature of allowing students time to reflect and to incubate on novel problems by providing a challenge time of any three weeks, during a specified six week period, in which to answer six novel problems. The time component of the Challenge was considered vital, since it was more likely to provide students with the opportunity to demonstrate cognitive and non-cognitive forms of reasoning. Moreover the notion of novelty in the problem-solving context was also critical. Much problem solving that takes place in mathematics classrooms is typically not novel but the Mathematics Challenge afforded the opportunity that the problems the students were to encounter were more likely to be new and different to them. Hence creative thinking and cognitive and non-cognitive ways of reasoning were more likely to be expressed. As Getzels (1975) stated "… the creativity of a solution depends on the creativity of the problem being solved."(p.84). Finally, because the Australian Mathematics Challenge was a national event access to a wide ranging sample from a broad cross section of the community was possible.

It was against the background of this national problem solving challenge event that a self-report instrument was designed in an attempt to measure two kinds of reasoning in creative problem solving.

Designing the Systems of Reasoning Questionnaire (SRQ)

In designing an instrument, which might be used to tap into cognitive and non-cognitive forms of reasoning a search of the relevant literature drew evidence from neurobiology, empirical psychology, popular psychology and philosophy. However for the confines of this paper, discussion will be restricted to evidence drawn predominantly from the fields of empirical and popular psychology.

Two Systems of Reasoning

Popular Psychology

Popular psychology has for some time now talked about two kinds of thinking or reasoning. The first kind has been popularly termed "computer thinking" and the other "free-flowing" or "transmitter thinking" (Carlson and Bailey 1997; Carlson 1998). Computer thinking has been characterised as effortful, linear, active, memory based and memory bound, conditional on emotions, predictable, and task detailed. Free-flowing thinking on the other hand has been characterised as passive, non-linear, selective of memory, employing deeper feelings, spontaneous, creative, inspired, visionary and big picture oriented.

Computer thinking has been popularly described as the kind of thinking which might be used by an individual in planning the steps needed to get from home to the airport in order to catch the aeroplane on time. It is thought to involve that part of the brain used in analysing, comparing, relating facts and making computations and is the kind of thinking generally taught in schools. Free-flowing thinking on the other hand has been popularly described as the kind of thinking that may occur when the individual does not know all the variables implicit to a problem, but having considered as many aspects as possible, sets aside the problem to incubate in the subconscious mind. Should the solution be required by a specified time, so long as the conscious mind has been primed then an answer should intuitively manifest itself in the time required (Carlson and Bailey 1997).

Hence it is from descriptions such as these that it may be possible to extrapolate that the popularly described computer thinking and free-flowing thinking are indicative to some degree of cognitive and non-cognitive forms of reasoning.

Empirical psychology

While the embracement of such popularly described forms of thinking should be treated with caution, the concept of two kinds of reasoning is well grounded in empirical psychology (Sloman 1996). Indeed the differentiation of two systems of reasoning have been cited as far back as Aristotle and can be found in the writings of James (1890/1950), Piaget (1926), Vygostsky (1934/1987) and Bruner (1986). While the configuration of the two systems may have shifted emphasis throughout history, with some psychologists arguing for one type of thinking in preference for another (eg. parallel processing through associative pathways versus symbol processing through serial analysis (Baer 1993)), others have argued for the dual existence of both systems of thinking (Neisser 1963; Johnson Laird 1983).

One advocate who conceives of the mind operating in both ways is Sloman (1996) and it is upon his empirical characterization of two forms of reasoning, that the Systems of Reasoning Questionnaire (SRQ) is based. (A summary of Sloman's characterisation of two systems of reasoning is provided in Table 1.)

Instead of names such as free-flowing and computer thinking, Sloman terms the dual systems of reasoning as the "associative system" and the "rule-based system". In comparing the associative system with the rule-based system he explains:

> Associative thought *feels* like it arises from a different cognitive mechanism than does deliberate, analytical reasoning. Sometimes conclusions simply appear at some level of awareness, as if the

> mind goes off, does some work, and then comes back with a result, and sometimes coming to conclusion requires doing the work oneself, making an effort to construct a chain of reasoning. (p.3, Sloman's italics)

Table 1. A table taken from page 7 of Sloman, S. A. (1996). The Empirical Case for Two Systems of reasoning. Psychological Bulletin, 119(1) 3-22 is shown.

TWO FORMS OF REASONING

Table 1
Characterization of Two Forms of Reasoning

Characteristic	Associative system	Rule-based system
Principles of operation	Similarity and contiguity	Symbol manipulation
Source of knowledge	Personal experience	Language, culture, and formal systems
Nature of representation		
Basic units	Concrete and generic concepts, images, stereotypes, and feature sets	Concrete, generic, and abstract concepts; abstracted features; compositional symbols
Relations	(a) Associations (b) Soft constraints	(a) Causal, logical, and hierarchical (b) Hard constraints
Nature of processing	(a) Reproductive but capable of similarity-based generalization (b) Overall feature computation and constraint satisfaction (c) Automatic	(a) Productive and systematic (b) Abstraction of relevant features (c) Strategic
Illustrative cognitive functions	Intuition Fantasy Creativity Imagination Visual recognition Associative memory	Deliberation Explanation Formal analysis Verification Ascription of purpose Strategic memory

Using an example from arithmetic to illustrate the difference, Sloman continues:

> Given an arithmetic problem such as figuring out change at the cash register, sometimes the answer springs to mind associatively, and sometimes a person has to do mental arithmetic by analyzing the amounts involved and operating on the resultant components as taught in school. (p.3)

In more precise discussion based on empirical evidence, Sloman characterises associative reasoning as operating reflexively, through a kind of statistical description of the environment in which similarity between problem elements is used to draw inferences. Such inferences may be drawn from general knowledge in the form of images, templates and stereotypes. Rule-based reasoning on the other hand operates in a causal-mechanical manner and is interpreted through different kinds of logical and hierarchical structures.

Notably, Sloman identifies a number of cognitive functions which are illustrative of the expert capacity of each reasoning system. The descriptions of many of these functions were used in framing statements for the SRQ. Intuition, creativity, imagination, fantasy, visual recognition and associative memory are the cognitive functions, which typify associative reasoning. Deliberation, explanation, formal analysis, verification, ascription of purpose and strategic memory are the cognitive functions, which typify rule-based reasoning. (Refer items 19, 20, 27 and 29 to 37 for some examples). Furthermore the source of knowledge in associative reasoning is derived from personal experience while that of rule-based reasoning is derived from formal elements in the culture. Thus the characterization of a student's knowledge source was also useful in formulating items for the SRQ and thereby the possible identification of the system of reasoning being used. (Items 21, 23 and 24 are cases in point.)

It should be pointed out that Sloman does not suggest in his findings that one style of reasoning exists in isolation from the other. Indeed they frequently overlap. Individuals may use both forms of reasoning in solving a problem even to the extent that conflicting or different answers may result. Drawing upon empirical data Sloman (1996) shows that individuals may simultaneously believe two contradictory answers to the same reasoning problem to demonstrate this point. Consequently identifying and classifying the two reasoning systems is not without its difficulties. Hence, to the degree in which individuals may use both forms of reasoning at the same time, some items in the SRQ may be characterized as drawing upon both rule-based and associative forms of reasoning, although in most instances there is a tendency to favour one form over another. (Items, 44, 45, 48 and 49 are examples of this.)

Sloman goes on to explain that each type of reasoning need not be exclusive to a given problem domain but that each mode may interact with the other contributing different computational resources to the problem at hand. Indeed because a given type of reasoning cannot be identified by a given problem domain, determining which system is responsible for a given response is not always possible. However, as a general rule the “contents of awareness”(p.6) Sloman (1996) states can be used as an indicator. Should an individual solve a problem with little or no awareness of the process used in the solution but be conscious only of the result then the response is likely to have been produced by the associative system. Thus a number of items were developed which sought to probe a student’s content of awareness and thus give a likely indication of which system of reasoning is being used. (Items 15,16, 17 and 18 are cases in point).

A table documenting the forty-five statements developed for the Systems of Reasoning Questionnaire (SRQ) together with the source of the idea for each statement is shown in Table 2. In addition the form of reasoning, be it rule-based or associative, which each statement was intended to measure is also given. The majority of these statements may be mapped to one of the five categories (viz: *Principles of Operation, Source of knowledge, Nature of Representation, Nature of Processing and Illustrative Cognitive Function*) outlined by Sloman in Table 1. In addition the category “contents of awareness” was added for emphasis, although statements in this category may be mapped to *Nature of Processing*. Furthermore ten items were also formulated which related to the four stage model of creative problem solving elaborated by Russ (1993). It was hoped that some pattern could be found linking the stages of creative problem solving with the form of reasoning used.

Procedure

Pilot Sample

A sample of 114 students from across four metropolitan private schools ranging from year 7 to year 10 was used in this pilot study. Of this sample, 56 students were male and 58 female. Subjects in each school were selected on the basis of their entry into the 2000 Australian Mathematics Challenge and on their willingness to be involved in the study. Participation in the study required that the student complete the questionnaire twice, once for each of the two problems selected, one in *number* and one in *space.* Thus although the challenge comprised six novel problems, students involved in the study answered the SRQ on only two designated problems.

Table 2. A table documenting the origin of ideas contained within each statement and the form of reasoning viz: associative and/or rule-based it was anticipated to measure in SRQ.

Item	Statement	Source of Idea; Author	Forms
15	I got answer first, then thought out mathematical reasons for it being correct	Contents of awareness; Sloman 96	Associative
16	I got the answer by using mathematical steps right from the beginning	Contents of awareness; Sloman 96	Rule Based
17	I got the answer but then had trouble explaining how I arrived at it	Contents of awareness; Sloman 96	Associative
18	I just knew how to get answer without using mathematical reasoning first	Contents of awareness; Sloman 96	Associative
19	I used my imagination in solving this problem	Cognitive function; Sloman 96	Associative
20	I used my intuition (followed a gut feeling) solving this problem	Cognitive function; Sloman 96	Associative
21	I used things I learned in schools while solving this problem	Source of knowledge; Sloman 96	Rule Based
22	The “answer” or “How to get the answer” suddenly came into my head while I was working on this problem	Nature of processing; Sloman 96	Associative
23	I associated this problem with things I experienced outside of school	Source of knowledge; Sloman 96	Associative
24	I associated this problem with images/pictures/diagrams I have seen	Source of knowledge; Sloman 96	Associative
25	I saw spatial/visual patterns in my mind	Nature of representatn; Sloman 96	Associative
26	I recognized number patterns in solving this problem	Nature of representatn; Sloman 96	Rule Based
27	I felt that I related to /connected with the patterns in this problem	Cognitive function; Sloman 96	Associative
28	I had a sense of number size in this problem	Nature of representatn; Sloman 96	Associative
29	I organized my reasoning /thinking in a strategic way	Cognitive function; Sloman 96	Rule Based
30	I recalled specific maths facts in solving this problem	Cognitive function; Sloman 96	Rule Based
31	I used a sequence of logical steps in this problem	Cognitive function; Sloman 96	Rule Based
32	I used a strategy/procedure I learned in class to solve this problem	Cognitive function; Sloman 96	Rule Based
33	I used specific formulae in solving this problem	Cognitive function; Sloman 96	Rule Based
34	I tried to verify/check that my answer was correct	Cognitive function; Sloman 96	Rule Based
35	I let my mind go free, thinking of any possibility solving this problem	Cognitive function; Sloman 96	Associative
36	I drew upon all mental resources/parts of me to solve this problem	Cognitive function; Sloman 96	Associative
37	I tried to be inventive in solving this problem	Cognitive function; Sloman 96	Associative
38	I developed a feeling about correctness of my solution before I checked it	Nature of processing; Sloman 96	Associative
39	I thought of how to get the answer to this problem while I was doing something else (eg working another problem, riding a bike)	Nature of processing; Sloman 96	Associative
40	I thought of how to get answer to problem when I woke up in morning	Nature of processing; Sloman 96	Associative
41	I tried a lot of different ways until I found the right one	Cognitive function; Sloman 96	Rule Based
42	Thought mainly with specific and exact numbers	Nature of representatn; Sloman 96	Rule Based
43	Thought mainly with approximate and general numbers	Nature of representatn; Sloman 96	Associative
44	Thought with both exact and approximate numbers	Nature of representatn; Sloman 96	Ass. & Rb
45	Thought with pictures/diagram (ie spatially) as well as with words	Nature of representatn; Sloman 96	Ass. & Rb
46	Thought more with words/symbols than with pictures/diagrams	Nature of representatn; Sloman 96	Rule Based
47	Thought more with pictures/diagrams than with words/symbols	Nature of representatn; Sloman 96	Associative
48	Thought with pictures & words/symbols at the same time	Nature of representatn; Sloman 96	Ass. & Rb
49	Often alternated between pictures and words	Nature of representatn; Sloman 96	Ass. & Rb
50	I carefully read the problem more than once	Creative PS Preparation; Russ 93	Rule Based
51	I took time to understand what the problem was asking	Creative PS Preparation; Russ 93	Rule Based
52	I had several attempts at finding a solution before I gave up for a time and came back to it later	Creative PS Incubation; Russ 93	Associative
53	I played around and explored the problem for a while	Creative PS Preparation; Russ 93	Rule Based
54	I spent time reflecting on how to solve the problem	Creative PS Incubation; Russ 93	Rule Based
55	I followed a feeling/hunch about what to do	Creative PS Illumination; Russ 93	Associative
56	I set the problem aside for a time	Creative PS Incubation; Russ 93	Associative
57	Having set aside the problem for a time I found the solution suddenly popped into my mind	Creative PS Illumination; Russ 93	Associative
58	The solution occurred to me after I had thought consistently for a time	Creative PS Illumination; Russ 93	Rule Based
59	I checked that my solution was correct	Creative PS Verification; Russ93	Rule Based

Instructions to Participants

Because entrants in the Mathematics Challenge had a period of three weeks in which to answer six challenge problems, participants were instructed to complete the SRQ as soon as practicable after completion of the relevant problem in order to facilitate accuracy and aid their responding. With regard to these instructions the scale devised for each of the 45 SRQ statements on which participants were invited to respond, comprised three points. These were True, Not Sure and False. The selection of a trichotomous scale was done for simplicity in order to keep the amount of missing data to a minimum (Keeves 1966) while at the same time providing a central category for participants who genuinely wished to indicate uncertainty on the grounds that the extreme category did not apply in its entirety. In addition it was felt that having fewer choices would be less confusing for the younger students in the sample, alleviating the dilemma associated with Likert scales of five points or more, in which some individuals tend to answer in the extremes while others routinely avoid them (Kline 1993). Further should it be necessary to unpack the responses amongst participants who responded with the "not sure" category (viz: "not sure" because they did not understand the statement or "not sure" because they were genuinely uncertain in their answer) the Hyperbolic Cosine Model (HCM) for unfolding responses could be used. (Andrich and Luo 1993, Andrich 1996). In the subsequent analysis, a score of two was assigned for a response of true, one for uncertain and zero for false.

Analysis Overview

The data collected in this pilot study were subjected to principal-components factor analysis (i.e. exploratory factor analysis) using SPSS (Version 10.0) rather than the more powerful confirmatory factor analysis for several reasons. Firstly, since research into the identification and description of cognitive and non-cognitive forms of reasoning during creative problem solving remains largely unoperationalised and the instrument devised in this study for measuring two forms of reasoning is in its infancy, a procedure which allowed the data to determine the underlying factor model was to be preferred in the first instance, over one that dictated what the model should be. As Carroll (1983) explains:

> Generally, exploratory methods are to be preferred for establishing the probable factorial composition and structure of a set of variables; confirmatory methods can then be applied to establish or test their significance. (p. 15)

Thus use of the exploratory factor analysis procedure in this study, was seen as a tool for theory building (Bryant and Yarnold 1998), which may later be put to the test using confirmatory factor analysis. Secondly the size of the data sample (N= 114) meant that conditions for using confirmatory factor analysis were less than optimal (Gustafsson and Balke 1993). Confirmatory factor analysis will be employed in the follow up study of the SRQ using a sample of at least N=400.

Initial Analysis

The exploratory factor analysis of the SRQ proceeded in several distinct stages. First because the SRQ was answered on two separate problems one in *number* and one in *space* separate analyses were conducted for each problem to determine any similarities and or differences in the formation of factors and to assess whether the data could legitimately be combined to yield a larger number of observations. Following the general principle of a *subjects–to-variables* (STV) ratio of approximately five or greater (Bryant and Yarnold 1998) these initial analyses were conducted on 27 items out of a possible 45 from the SRQ. Each data set was subjected to a principal components extraction followed by a varimax rotation, yielding three distinct but common factors. These factors were recognizable in both *number* and *space* as the *Systematic* Factor, the *Free-flowing* Factor and the *Feeling* Factor. Although there are differences in the distribution and loading of some items, the factors were found by inspection to be sufficiently stable (i.e. similar and constant) across problem type for the data sets to be combined for subsequent analyses. Furthermore since the focus of the study is on the processes of creative problem solving rather than the influence of problem type and that these processes are hypothesized to be generic across problem situations the final analyses were conducted on the combined space and number data and it is upon the combined data that the paper reports. The results of these initial rotated factor solutions are located in Appendices 1 and 2.

Final Analysis

Following the initial exploratory factor work, principal components factor analyses were now conducted on 45 items in the SRQ using the observations from the combined space and number data (N = 198, listwise deletion of data). These were followed by a varimax rotation to achieve simple structure, assisting the resolution of factors that would help build a model of creative problem solving involving cognitive and non-cognitive forms of reasoning. The results of the rotated factor pattern are given in Tables 3 and 4.

RESULTS AND DISCUSSION

Factor Structure

A five-factor structure accounting for 38 percent of the total variance was obtained. Although initial principal components analysis yielded 13 components whose eigenvalues were greater than one, accounting for 62 percent of the variance, five of the 13 components had eigenvalues greater than two. A scree test (Cattell 1966), performed in order to determine the number of factors, clearly indicated the presence of five distinct factors. A varimax rotation of five factors was undertaken and of the 45 items under consideration in the SRQ, 43 had loadings of 0.30 and above. Further a six-factor extraction carried out as a precautionary measure, yielded un-meaningful data.

With the exception of two items (viz: item 35 on factor 1 and item 46 on factor 3) all items within each scale are positive. However taking into consideration the composition of all other items in each respective scale, the negative loadings for these items is not inconsistent with the interpretation of each scale. Nonetheless consideration as to whether these items are ultimately included will be undertaken upon further trailing. The items, which did not load according to the 0.3 criteria, were item 27 (*I felt that I related to/connected with the patterns in this problem (0.27 on factor 5))* and item 43 (*Thought mainly with approximate and general numbers (0.26 on factor 4)).*

The reliability, for each factor (Kaiser and Caffrey 1965) is documented in Table 4, together with the percent variance explained by each factor, both before and after orthogonal rotation. Factors one, two and three display good reliability while factors four and five have satisfactory reliabilities.

Interpretation of Factors and the Formation of Scales

On the basis of the factorial structure arrived at, five scales corresponding to each of the five factors have been derived. These scales have been identified as:

- *Systematic* approach to reasoning
- *Strategic* approach to reasoning
- *Spatial/Verbal* approach to reasoning
- *Free-flowing* approach to reasoning
- *Feeling* approach to reasoning

Table 3. Principal Component Solution Rotated to Orthogonal Structure (loadings > 0.3)

Item	Statement	Fac1	Fac2	Fac3	Fac4	Fac5
A28	I had a sense of number size in this problem	0.57				
R29	I organized my reasoning /thinking in a strategic way	0.57				
R26	I recognized number patterns in solving this problem	0.56		-0.33		
R16	I got the answer by using mathematical steps right from the beginning	0.53				
R30	I recalled specific maths facts in solving this problem	0.53				
R31	I used a sequence of logical steps in this problem	0.52				
R33	I used specific formulae in solving this problem	0.51				
RA44	Thought with both exact and approximate numbers	0.47				
R21	I used things I learned in schools while solving this problem	0.46				
R42	Thought mainly with specific and exact numbers	0.45				
R32	I used a strategy/procedure I learned in class to solve this problem	0.38				
A35	I let my mind go free, thinking of any possibility solving this problem	-0.36				
R41	I tried a lot of different ways until I found the right one		0.62			
R51	I took time to understand what the problem was asking		0.61			
R53	I played around and explored the problem for a while		0.60			
R50	I carefully read the problem more than once		0.55			
A17	I got the answer but then had trouble explaining how I arrived at it		0.51			
A52	I had several attempts at finding a solution before I gave up for a time and came back to it later		0.50			-0.33
R58	The solution occurred to me after I had thought consistently for a time		0.47			
R54	I spent time reflecting on how to solve the problem		0.47			
AR 45	Thought with pictures/diagram (ie spatially) as well as with words			0.78		
A47	Thought more with pictures/diagrams than with words/symbols			0.77		
R46	Thought more with words/symbols than with pictures/diagrams			-0.61		
AR49	Often alternated between pictures and words			0.57	0.34	
A25	I saw spatial/visual patterns in my mind			0.56		
A24	I associated this problem with /images/pictures/diagrams I have seen			0.53		
AR48	Thought with pictures & words/symbols at the same time			0.33		
A23	I associated this problem with things I experienced outside of school				0.59	
A40	I thought of how to get answer to problem when I woke up in morning				0.59	
A57	Having set aside the problem for a time I found the solution suddenly popped into my mind				0.56	
A39	I thought of how to get the answer to this problem while I was doing something else (eg. working another problem, riding a bike)				0.56	
A36	I drew upon all mental resources/parts of me to solve this problem				0.38	
A56	I set the problem aside for a time		0.31		0.36	-0.33
R34	I tried to verify that my answer was correct		0.42			0.59
A55	I followed a feeling/hunch about what to do					0.59
R59	I checked that my solution was correct		0.33			0.59
A20	I used my intuition (followed a gut feeling) solving this problem					0.48
A19	I used my imagination in solving this problem	-0.34				0.47
A22	The "answer" or "How to get the answer" suddenly came into my head while I was working on this problem					0.45
A18	I just knew how to get the answer without using maths* reasoning first		-0.38			0.45
A38	I developed a feeling about correctness of my soln* before I checked it	0.35				0.41
AR15	I got answer first, then thought out maths* reasons for it being correct					0.38
A37	I tried to be inventive in solving this problem				0.31	0.36

Factor one, which corresponds to the scale for measuring *Systematic* approaches to reasoning, reflects the intent of a student who structures his thinking in a methodical and systematic way, who consciously and deliberately seeks to recall relevant information (e.g. number facts, procedures) to solve a problem and who progresses his thinking in a linear, logical and effortful way.

Table 4. Summary information for each scale

Factor No.	Scale Name	Nos. of Items	Reliability Kaiser Caffrey	% Variance explained before rotation	% Variance explained after rotation
One	Systematic	12	0.75	11.09	8.34
Two	Strategic	8	0.70	8.39	7.68
Three	Spatial/Verbal	7	0.76	6.98	7.52
Four	Free-flowing	6	0.65	6.00	7.23
Five	Feeling	10	0.68	5.13	6.83

Systematic approaches to reasoning are largely the kinds of thinking processes, which have generally been emphasized in mathematics curricula as taught in schools (Lovitt 1999). While these kinds of thinking strategies are critical to the documentation of a solution and ultimate success, of themselves, they may yet be found to be insufficient in fruitfully solving a truly novel problem.

Factor two, which has been identified as the *Strategic* approach to reasoning, reflects the tactics of a student who employs elements of the four stage model of creative problem solving to facilitate their thinking including conscious and deliberate preparation and non-conscious and semi-conscious incubation, to ultimately resolve a solution.

While in reality the four stages of the creative problem solving process are probably not so ordered as the original proponents espoused (Wallas 1925, Hadamard 1945), it is the ability to shift back and forth between the stages, sometimes drawing upon cognitive reasoning, at other times non-cognitive forms of reasoning,(i.e. preparation and incubation) that may prove vital in the creative problem solving process (Russ 1993).

Factor three, which corresponds to the scale for measuring *Spatial/verbal* approaches to reasoning, mirrors the nature of thinking a student may employ be it spatial, verbal, or both spatial and verbal, in seeking a solution to a problem. The negative loading for the statement *thought more with words/symbols than with pictures/diagrams* however, would seem to indicate that the factor is aligned more strongly with the spatial, rather than verbal component of reasoning.

Recent neuro-biological evidence gathered on two systems of reasoning has identified two brain circuits used in mathematical thinking. One circuit is used for approximate arithmetic, the other for exact arithmetic. Interestingly the circuit associated with approximate arithmetic is located in a region strong in visual-spatial processing while that for exact arithmetic is located in a region strong in linguistic processing (Dehaene et al 1999). Thus this factor may be tapping into both visual spatial and linguistic circuits of reasoning.

Factor four, which has been identified as the *Free-flowing* approach to reasoning, mirrors the intent of a student who has employed non-conscious or semi-conscious forms of reasoning, to solve a problem such as that undertaken during incubation. Since the very action of non-conscious reasoning is un-conscious, the only indication that such a form of reasoning has taken place is by a positive response to some indicative behaviours which may reflect the process, such as thinking of the answer while doing something else, or arriving at an answer upon first waking in the morning. In addition, an individual who draws upon this approach to reasoning may well access, stored information, which is more global in its origin.

Of interest is the notion proposed by Torrance and Rockstein (1988) that the inclusion of incubation time during problem solving is critical to the putting together of processes, both cognitive and non-cognitive, needed for whole brain functioning and the creative solution of problems.

Factor five, which corresponds to the scale for measuring *Feeling* approaches to reasoning, reflects the individual who has followed a feeling or hunch about what to do and tries to verify or check what they have done. In addition a feeling about the quality and correctness of the solution may be present before the answer is checked. Individuals who utilize this approach to reasoning may be synthesizing information drawn from non-conscious forms of reasoning with that of conscious deliberate reasoning.

Interestingly in trying to clarify the difference between intelligence and creativity Sternberg and O'Hara (1999) cite Shouksmith (1973) as making a distinction between the rightness and goodness of a response. In Shouksmith's view judging the rightness of a response is an attempt to measure logical reasoning while judging the goodness of a response is a measure of creativity. The intersection of these two would yield responses that are both right and good.

Accessing Two Underlying Dimensions

On another level of analysis it is also possible to investigate the structure of the five factors using a system of *a priori* classification, undertaken prior to statistical appraisal as a means of identifying a statement as being rule-based or associative in its approach to reasoning. Viewing the labels associated with each of the items permits access to this system of *a priori* classification for additional analysis.

Items that were originally classified as demonstrating rule-based reasoning as indicated by their description of behaviour have the prefix R in front of the item number. Those items that were originally classified as demonstrating associative reasoning as indicated by their description of behaviour have the prefix A in front of the item number. Those items representing both rule-based and associative reasoning have the letters AR or RA as a prefix to the item number. Hence an examination of the factors with this classification in mind yields some interesting findings. A break down of the number and nature of items by factor is given in Table 5.

Table 5. Table showing composition of each factor using a priori *classification of items.*

				Anticipated Form of Reasoning		
Factor No.	Scale Name	Nos. of Items	Nos. of items classified Rule-based	Nos. of items classified Associative	Nos. of items classified both Rule-based & Associative	Dominant classification
One	Systematic	12	9	1	1	Rule-based
Two	Strategic	8	6	2	0	Rule-based
Three	Spatial/verbal	7	1	3	3	Both /Assoc
Four	Free Flowing	6	0	6	0	Associative
Five	Feeling	10	2	7	1	Associative
Total		43	18	19	5	

The *Systematic* factor and *Strategic* factor appear to be comprised of items that were classified largely as rule-based in their approach to reasoning. The *Free-flowing* factor and the *Feeling* factor appear to be comprised of items that were classified largely as associative in their approach to reasoning. Interestingly the *Spatial/verbal* factor comprises items that were classified as involving both rule-based and associative forms of reasoning while at the same time having a number of statements that were classified as being associative. This would appear to be consistent with the tendency of the factor to be more closely aligned with the spatial rather than verbal component of reasoning.

While this *a priori* method of classifying items may not be excessively generalized to indicate the nature of the underlying dimension within each scale without a good deal of further testing, the evidence accumulated thus far is compelling in its consistency.

CONCLUSION

In conclusion it would appear that cognitive and non-cognitive forms of reasoning may be identified, described and measured in the novel mathematics problem solving context through the use of a self report questionnaire (viz: Systems of Reasoning Questionnaire SRQ). Sloman's (1996) characterization of two forms of reasoning, together with the four stage model of creative problem solving have provided a useful framework by which this can be done. Indeed cognitive and non-cognitive measurements gathered from preliminary data indicate that success in creative problem solving is significantly correlated with the *Feeling* approach to reasoning (Aldous 2001, in press).

While the preliminary work outlined in this paper needs to be replicated with larger samples and tested with additional procedures (e.g. confirmatory factor analysis) the potential for accessing other elements of the teaching and learning system emanates from the administration of an instrument such as this to students undertaking a novel problem solving learning activity. As Biggs (2001) has pointed out, it is the interaction between the student, the task and the context that is vital in shaping future directions in learning and teaching and the students' approaches to reasoning and learning are “the barometers that tell how well the general system is working”.

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APPENDIX 1

Table 5. Principal component solution rotated to orthogonal structure derived from Space data N=100 (Loadings >= 0.3). Items in common with Number are indicated with #.

Item	Common	Statement	1	2	3
		Free Flowing Factor			
R32		I used a strategy/procedure I learned in class to solve this problem.	0.57		
A17		I got the answer but then had some trouble explaining how I arrived at it.	0.55		
A40	#	I thought of how to get the answer to this problem when I woke up in the morning.	0.54		
A37	#	I tried to be inventive in solving this problem.	0.54		
A36	#	I drew upon all mental resources/parts of me to solve this problem.	0.47		
A39	#	I thought of how to get the answer to this problem while I was doing something else (e.g. sitting in school, working another problem, riding a bike).	0.46		
A23	#	I associated this problem with things I experienced/learned outside of school..	0.43		
R21		I used things I learned in school while solving this problem	0.421		0.33
A35		I let my mind go free, thinking of any possibility when solving this problem.	0.42	-0.36	
A41	#	I tried a lot of different ways until I found the right one.	0.407		
		Systematic Factor			
A28		I had a sense of number size in this problem.		0.74	
R26		I recognised number patterns in solving this problem.		0.61	
R33	#	I used specific formulae in solving this problem.		0.56	
R16	#	I got the answer by using mathematical steps right from the beginning.		0.53	
A24		I associated this problem with things/images/pictures/diagrams etc. I have seen.		0.44	
R30	#	I recalled specific maths facts in solving this problem	0.39	0.44	
R29	#	I organised my reasoning/thinking in a strategic way.		0.40	0.31
		Feeling Factor			
R34	#	I tried to verify/check that my answer was correct.			0.56
A38	#	I developed a feeling about the correctness of my solution before I checked it.	-0.35	0.41	0.51
A19		I used my imagination in solving this problem		-0.49	0.51
A25		I saw spatial/visual patterns in my mind			0.50
A18	#	I just knew how to get the answer without using mathematical reasoning first.			0.46
A20	#	I used my intuition in solving this problem			0.44
R31		I used a sequence of logical steps in this problem			0.43
A27	#	I felt that I related to/connected with the patterns in this problem.			0.40
A22	#	The “Answer” or “How to get the answer” suddenly came into my head while I was working on this problem.			0.39

*Table 6. Table showing summary details of the three factored scales based on **Space** data*

Factor No.	Scale Name	Nos. of Items	Nos. of Common Items	Reliability Kaiser-Caffrey	% Variance explained before rotation	% Variance explained after rotation	Dominant classif-ication
One	Free-flowing	10	6	0.67	13.27	10.73	Associative
Two	Systematic	7	4	0.69	9.00	10.58	Rule-based
Three	Feeling	9	5	0.60	8.48	9.44	Associative

APPENDIX 2

*Table 7. Principal component solution rotated to orthogonal structure derived from **Number** data N=102. (Loadings >= 0.3). Items in common with Space are indicated with #.*

Item	Common	Statement	1	2	3
		Systematic Factor			
R16	#	I got the answer by using mathematical steps right from the beginning.	0.64		
R31		I used a sequence of logical steps in this problem.	0.64		
R29	#	I organized my reasoning/thinking in a strategic way.	0.61		.0.40
R30	#	I recalled specific maths facts in solving this problem.	0.60	0.34	
R32		I used a strategy/procedure I learned in class to solve this problem.	0.52	0.45	
A35		I let my mind go free, thinking of any possibility when solving this problem.	-0.52	0.34	
A17		I got the answer but then had trouble explaining how I arrived at it.	-0.49		
R21		I used things I learned in school while solving this problem.	0.48	0.32	
R33	#	I used specific formulae in solving this problem	0.47		
		Free-flowing Factor			
A39	#	I thought of how to get the answer to this problem while I was doing something else (e.g. sitting in school, working another problem, riding a bike)		0.58	
A23	#	I associated this problem with things I experienced/learned outside of school.		0.57	
A37	#	I tried to be inventive in solving this problem.		0.54	0.36
A24		I associated this problem with things/images/pictures/diagrams etc. I have seen.		0.53	
A40	#	I thought of how to get the answer to this problem when I woke up in the morning.		0.53	
A36	#	I drew upon all mental resources/parts of me to solve this problem.		0.50	
A25		I saw spatial/visual patterns in my mind.		0.48	
A19		I used my imagination in solving this problem	-0.39	0.48	
A41	#	I tried a lot of different ways until I found the right one.		0.35	
		Feeling Factor			
R34	#	I tried to verify/check that my answer was correct.			0.66
A28		I had a sense of number size in this problem			0.66
AR15		I got the answer first and then thought out the mathematical reasons for it being correct.			0.56
A38	#	I developed a feeling about the correctness of my solution before I checked it.			0.55
R26		I recognized number patterns in solving this problem.	0.30		0.54
A18	#	I just knew how to get the answer without using mathematical reasoning first.			0.44
A22	#	The "Answer" or "How to get the answer" suddenly came into my head while I was working on this problem.			0.44
A27	#	I felt that I related to/connected with the patterns in this problem.			0.41
A20	#	I used my intuition (followed a gut feeling) solving this problem		0.299	0.29

*Table 8. Table showing summary details of the three factored scales based on **number** data*

Factor No.	Scale Name	Nos. of Items	Nos. of Common Items	Reliability Kaiser-Caffrey	% Variance explained before rotation	% Variance explained after rotation	Dominant classi-fication
One	Systematic	9	4	0.76	16.23	12.12	Rule-based
Two	Free-flowing	9	6	0.69	11.13	11.75	Associative
Three	Feeling	8	5	0.68	7.56	11.11	Associative